A capable neural network model for solving the maximum flow problem

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This paper presents an optimization technique for solving a maximum flow problem arising in widespread applications in a variety of settings. On the basis of the Karush–Kuhn–Tucker (KKT) optimality conditions, a neural network model is constructed. The equilibrium point of the proposed neural network is then proved to be equivalent to the optimal solution of the original problem. It is also shown that the proposed neural network model is stable in the sense of Lyapunov and it is globally convergent to an exact optimal solution of the maximum flow problem. Several illustrative examples are provided to show the feasibility and the efficiency of the proposed method in this paper.

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1. Introduction

The constrained maximum flow problem is to send the maximum flow from a source to a sink in a directed capacitated network where each arc has a cost and the total cost of the flow cannot exceed a budget. This problem is similar to some variants of classical problems such as the constrained shortest path problem, constrained transportation problem, or constrained assignment problem, all of which have important applications in practice. The constrained maximum flow problem itself arises in a wide variety of scientific and engineering applications including logistics, telecommunications and computer networks.

In many engineering and scientific applications, real-time online solutions of the maximum flow problems are often desired. However, traditional algorithms [1–7] may not be applicable for digital computers since the computing time required for a solution is greatly dependent on the dimension and the structure of the problem and the complexity of the algorithm used. One promising approach for handling on-line applications is to employ recurrent neural networks based on circuit implementation. The main idea of the neural network approach for an optimization problem is to construct a nonnegative energy function and establish a dynamic system that represents an artificial neural network. The dynamic system is usually in the form of first-order ordinary differential equations. Furthermore, it is expected that the dynamic system will approach its static state (or equilibrium point), which corresponds to the solution for the underlying optimization problem, starting from an initial point. An important requirement is that the energy function decreases monotonically as the dynamic system approaches an equilibrium point. Because of the dynamic nature and the potential of electronic implementation, neural networks can be implemented physically by designated hardware such as application-specific integrated circuits, where the computational procedure is truly distributed and parallel. Therefore, the neural